

# 工程數學 Midterm #2 Dec, 6 2022

|   |   |
|---|---|
| 1. 5pt<br>$L\{(a - bt)^2\},$<br>a and b are constant            | 7. 10pt<br>$y'' + 2y' - 3y = 0,$<br>$y(2) = -3, \quad y'(2) = -5$                                       |
| 2. 10pt<br>$L^{-1}\left\{\frac{6s + 7}{2s^2 + 4s + 10}\right\}$ | 8. 10pt<br>$L^{-1}\left\{\frac{2}{s^2 + \frac{s}{3}}\right\}$   |
| 3. 5pt<br>$L\{10e^{-3t} - 5t + 8\}$                             | 9. 10pt<br>$y'' + 4y' + 6y = 1 + e^{-t},$<br>$y(0) = 0, \quad y'(0) = 0$                                |
| 4. 10pt<br>$L^{-1}\left\{\frac{0.2s + 1.4}{s^2 + 1.96}\right\}$ | 10. 10pt<br>$L^{-1}\left\{\frac{5s}{(s - 2)^2}\right\}$   |
| 5. 10pt<br>$L^{-1}\left\{\frac{11 - s}{s^2 - 2s - 3}\right\}$   | 11. 10pt<br>$L^{-1}\frac{1}{s^3 + s}$<br>$x^2y'' = D_t(D_t - 1)y$<br>$x^3y''' = D_t(D_t - 1)(D_t - 2)y$ |
| 6. 10pt<br>$L^{-1}\left\{\frac{s}{s^2 + 4s + 5}\right\}$        |   |

## ● Laplace 轉換

|   | $f(t)$                         | $\mathcal{L}(f)$              |    | $f(t)$                 | $\mathcal{L}(f)$                    |
|---|--------------------------------|-------------------------------|----|------------------------|-------------------------------------|
| 1 | 1                              | $1/s$                         | 7  | $\cos \omega t$        | $\frac{s}{s^2 + \omega^2}$          |
| 2 | $t$                            | $1/s^2$                       | 8  | $\sin \omega t$        | $\frac{\omega}{s^2 + \omega^2}$     |
| 3 | $t^2$                          | $2!/s^3$                      | 9  | $\cosh at$             | $\frac{s}{s^2 - a^2}$               |
| 4 | $t^n$<br>( $n = 0, 1, \dots$ ) | $\frac{n!}{s^{n+1}}$          | 10 | $\sinh at$             | $\frac{a}{s^2 - a^2}$               |
| 5 | $t^a$<br>( $a$ positive)       | $\frac{\Gamma(a+1)}{s^{a+1}}$ | 11 | $e^{at} \cos \omega t$ | $\frac{s-a}{(s-a)^2 + \omega^2}$    |
| 6 | $e^{at}$                       | $\frac{1}{s-a}$               | 12 | $e^{at} \sin \omega t$ | $\frac{\omega}{(s-a)^2 + \omega^2}$ |

First Shifting Theorem,  $s$ -Shifting

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

$$e^{at}f(t) = \mathcal{L}^{-1}\{F(s-a)\}$$

Laplace Transform of Derivatives

$$\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$$

$$\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0)$$

$$\mathcal{L}(f^{(n)}) = s^n\mathcal{L}(f) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$$

Laplace Transform of Integral 積分轉換

$$\mathcal{L}\left\{\int_0^t f(\tau) d\tau\right\} = \frac{1}{s} F(s), \quad \text{thus} \quad \int_0^t f(\tau) d\tau = \mathcal{L}^{-1}\left\{\frac{1}{s} F(s)\right\}$$