

計算小錯誤無部份給分

答案卷請務必在每一張寫上姓名、學號，沒寫不算分。Solve the ODEs:

Total 100 points; 10 points each.

$$1. \frac{dy}{dx} = \frac{xy+2y-x-2}{xy-3y+x-3}$$

$$2. \frac{dy}{dx} + 2xy^2 = 0$$

$$3. \frac{dy}{dx} = y^2 - 4$$

4. $y' + 2y = 0$ has an integrating factor (積分因子) e^{2x} , solve the ODE

5. $x \frac{dy}{dx} + 2y = 3$ has an integrating factor (積分因子) $I(x)$

5-1. Find $I(x)$

5-2. Solve the ODE:

$$6. y'' - y' - 6y = 0$$

$$7. y^{(4)} - 2y'' + y = 0$$

$$8. y'' + 16y = 0, \quad y(0) = 2, \quad y'(0) = -2$$

$$9. y'' - y' = -3$$

$$10. x^2 y'' - 2y = 0$$

公式表

求 y_h :

Auxiliary equation: $m^2 + am + b = 0$ 根為: $\frac{-a \pm \sqrt{a^2 - 4b}}{2}$

$\sqrt{a^2 - 4b}$ 解有三種可能:

<1>相異實根: m_1, m_2

ODE 通解為: $y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

<2>實數重根: m

ODE 通解為: $y = (c_1 + c_2 x) e^{mx}$

<3>複數根 (共軛虛根): $\alpha \pm \omega i$

ODE 通解為: $y = e^{\alpha x} (A \cos \omega x + B \sin \omega x)$

求 y_p :

Undetermined coefficients method (未定係數法):

$R(x)$	y_p 假設型
k	A
e^{ax}	Ae^{ax}
$\cos bx$ 或 $\sin bx$	$A \cos bx + B \sin bx$
x^n	$A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x^1 + A_0$
cx^n	$A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x^1 + A_0$
$x^n e^{nx}$	$e^{nx} (A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x^1 + A_0)$
$cx^n e^{nx}$	$e^{nx} (A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x^1 + A_0)$
$x^n \cos bx$ 或 $x^n \sin bx$	$(A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x^1 + A_0) \cos bx + (B_n x^n + B_{n-1} x^{n-1} + \dots + B_1 x^1 + B_0) \sin bx$

註: a, b, c, k, n, A, B 為常數

- Cauchy-Euler equation:

令 $t = \ln x$ (以 t 取代 x)

$$xy' = D_t y$$

$$x^2 y'' = D_t (D_t - 1)y$$

$$x^3 y''' = D_t (D_t - 1)(D_t - 2)y$$