

ID: _____ Name: _____

滿分 20 分。答案若是有 Σ 符號，可以不用展開，作答方式如上課所示即可

[3%] (1) 已知 $F(s)$ 求出 $f(t)$

$$F(s) = \frac{2e^{5s}}{s^2 - 2s + 1}$$

[3%] (2) 已知 $F(s)$ 求出 $f(t)$

$$L^{-1} \left\{ \frac{-e^{-s}}{s(s+1)} \right\}$$

[4%] (3) 求出 $f(x)$ 的傅立葉級數

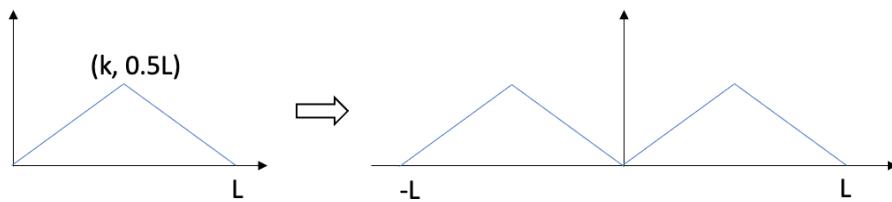
$$f(x) = \begin{cases} 1, & -1 < x < 0 \\ x, & 0 < x < 1 \end{cases} \quad (\text{週期為 } 2)$$

[3%] (4) 求出 $f(x)$ 的傅立葉級數

$$f(x) = x^2, \quad -1 < x < 1$$

[3%] (5) 求 $f(x) = 3x + 5\pi$ 當 $-\pi < x < \pi$ 且 $f(x + 2\pi) = f(x)$ 之傅立葉級數

[4%] (6) [半幅展開式] 求 $f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < L \end{cases}$ 的偶函數延伸



公式表:

	$f(t)$	$\mathcal{L}(f)$		$f(t)$	$\mathcal{L}(f)$
1	1	$1/s$	7	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
2	t	$1/s^2$	8	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
3	t^2	$2!/s^3$	9	$\cosh at$	$\frac{s}{s^2 - a^2}$
4	t^n ($n = 0, 1, \dots$)	$\frac{n!}{s^{n+1}}$	10	$\sinh at$	$\frac{a}{s^2 - a^2}$
5	t^a (a positive)	$\frac{\Gamma(a+1)}{s^{a+1}}$	11	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
6	e^{at}	$\frac{1}{s-a}$	12	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$

First Shifting Theorem, s -Shifting

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

$$e^{at}f(t) = \mathcal{L}^{-1}\{F(s-a)\}$$

Laplace Transform of Derivatives

$$\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$$

$$\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0)$$

$$\mathcal{L}(f^{(n)}) = s^n\mathcal{L}(f) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$$

Laplace Transform of Integral 積分轉換

$$\mathcal{L}\left\{\int_0^t f(\tau) d\tau\right\} = \frac{1}{s} F(s), \quad \text{thus} \quad \int_0^t f(\tau) d\tau = \mathcal{L}^{-1}\left\{\frac{1}{s} F(s)\right\}$$

Second Shifting Theorem; Time Shifting

$$\tilde{f}(t) = f(t-a)u(t-a) = \begin{cases} 0 & \text{if } t < a \\ f(t-a) & \text{if } t > a \end{cases}$$

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$$

$$f(t-a)u(t-a) = \mathcal{L}^{-1}\{e^{-as}F(s)\}$$

Examples for t -shifting by 1

$$1 \mapsto \frac{1}{s} \quad u(t-1) \mapsto e^{-s} \frac{1}{s}$$

$$t \mapsto \frac{1}{s^2} \quad (t-1)u(t-1) \mapsto e^{-s} \frac{1}{s^2}$$

$$t^2 \mapsto \frac{2}{s^3} \quad (t-1)^2 u(t-1) \mapsto e^{-s} \frac{2}{s^3}$$

$$\sin t \mapsto \frac{1}{s^2+1} \quad \sin(t-1)u(t-1) \mapsto e^{-s} \frac{1}{s^2+1}$$

傅立葉級數 (週期 $2L$)

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right)$$

- (a) $a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx$
- (b) $a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx \quad n = 1, 2, \dots$
- (c) $b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx \quad n = 1, 2, \dots$

Fourier Cosine Series (*even function, 偶函数*) (週期 $2L$)

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{L} x \quad (f \text{ even})$$

$$a_0 = \frac{1}{L} \int_0^L f(x) dx, \quad a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx, \quad n = 1, 2, \dots$$

Fourier Sine Series (*odd function, 奇函数*) (週期 $2L$)

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{L} x \quad (f \text{ odd})$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$$